Applying Resource Loading, Production & Learning Curves to Construction: A Pragmatic Approach

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Abstract

The purpose of this paper is to present some rules of thumb based on experience for the early planning of new civil and building construction work. In such construction, resource input (men, materials, equipment, etc.) is varied according to the planned timing and availability of the work. On a well-run site, this resource loading as well as its consequent output follows a distinctive pattern within relatively narrow limits for the whole of the job. Practical considerations why this should be so are presented.

Based on experience, this paper suggests first approximation profiles for both typical resource loading and progress S-curves, and shows that the difference could be due to the effects of learning. The basis for calculating the shape of the learning curve and how the application of this concept is limited on a construction site are described. The manner in which an alternative learning curve calculation can be more useful in tracking progress is demonstrated. The significance of these profiles and their relationships for improved planning and tracking of new construction work is suggested. An example of the output from a less-well managed project as compared to the planned S-curve is also included.

***Keywords**:* learning curve, productivity improvement, progress/production curve, resource loading.

Introduction

In order to optimize productivity on new facility construction, the input of resources including men, materials and equipment, is varied according to the planned timing and availability of the work. This applies on all but the smallest construction jobs where minimal crew size may limit flexibility. However, even on quite small “maintenance projects" this flexibility may be facilitated by managing the manpower levels over several concurrent assignments. This optimizing of productivity results in an initial period of build up, a period of peak loading, followed by a period of progressive demobilizing. This typical profile, or curve, when plotted cumulatively over time for a whole project, results in another typical curve in the shape of the letter “S”.

The purpose of this paper is to present some rules of thumb relating to these curves which have resulted from experience on new civil and building construction work. These rules of thumb suggest simple ways to draw first approximations for cumulative resource or production curves over the life of the project. The first relates to resource loading, i.e. men, materials, equipment or cash. A second relates to the consequent production output. The two curves are closely related and it is suggested that the difference can be accounted for by the effects of learning. The phenomenon of learning itself is also explored to show how it may be used for planning or tracking repetitive tasks on construction work.

These relationships have been found by the author to be most helpful for preliminary project planning, for checking the validity of proposed plans, or for analyzing the records of completed work. Since the author has used these techniques while employed variously by owners, developers, and general contractors, it is hoped that they will be seen as beneficial for anyone in similar positions.

In the following discussion, unless otherwise stated, the presumption is that the project is or will be "well run". For the definition of a well-run construction job, refer to Appendix 1.

Resource Loading (input)

Figure 1 shows an example of manpower loading for a profitable civil contract which was predominantly formwork and concrete placing.[1] It displays a histogram of the make up and total numbers in the production work force, week by week over a 38 week period. It also shows the progressive cumulative total, or actual manpower loading S-curve. As noted earlier, the general profiles of these curves appear to be quite typical (Christian 1991) whether the observations refer to a whole construction project, a sub-contract, an individual trade or a continuous construction activity of significant duration.

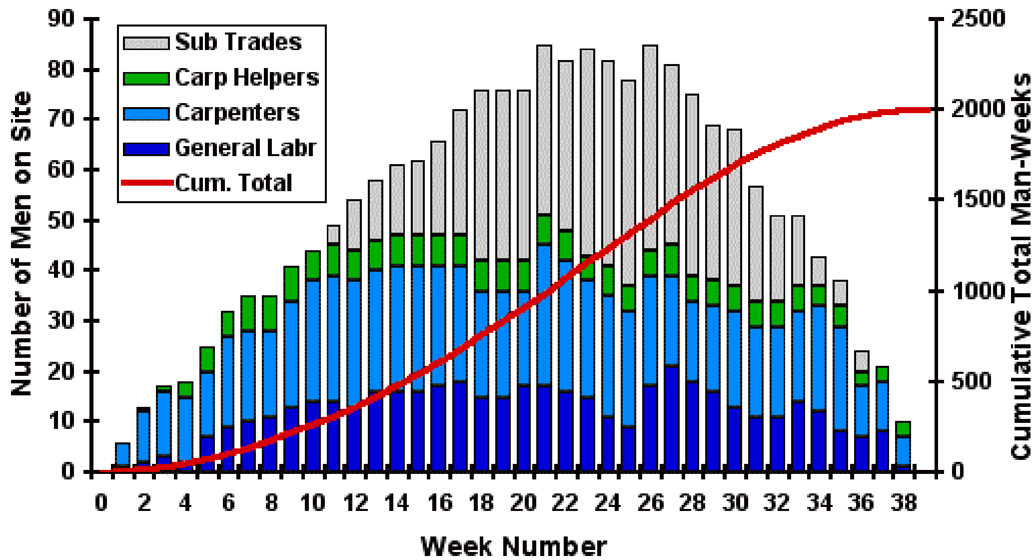

Figure 1 – Civil contract example of site manpower (predominantly) concrete work

The important points to note about the S-curve are that the initial part of the curve represents the "build-up", the central part of the curve is relatively "steady-state", or effectively a straight line slope, and the latter part of the curve represents a "run-down" which closely mirror-images the early part of the S-curve.

In order to understand the generality of the suggested rules of thumb that follow, it is instructive to recite the many practical reasons for the shapes of each of these three stages.

1 From the author's personal records of progress tracking.

In Stage 1 there is an accelerating build up of manpower because

- Access to the work has to be opened up from a zero start, with the result that the work itself becomes progressively more available.
- Necessary preliminary preparatory activities, including planning and understanding local conditions, as well as ordering of materials, etc., often require fewer people but more intensive supervision.
- Key people may be brought in to start the work, but supporting labour is recruited locally. The recruiting and selection of local labour itself takes time.
- With productive efficiency in mind, crews are added only as experience builds and the work becomes available to be performed.
- Further crews are added only as pressure builds to get the job done within the required time frame.

Stage 2 achieves a steady state because

- The working environment has reached optimum conditions for balanced performance and repetition.
- Physical limitations to the capabilities of the men and equipment provided is reached.
- Adding more labour or separate crews would over-crowd the working area and reduce productivity.
- The number of repetitions available from which the benefits of "productivity improvement" can be derived would be reduced.
- In either case the costs would be higher.
- Alternatively, if the work force is held at a lower level, the elapsed time to accomplish the work will be prolonged, with consequent higher overheads and, possibly, contract penalties to be faced.

These obvious trade-offs require careful management and balance.

In stage 3 almost the reverse of Stage 1 is true. Manpower is progressively reduced because

- The work begins to run out.
- The remaining work space not occupied by following trades, or owner occupation, runs the danger of becoming over crowded.
- Morale sometimes deteriorates as the end of the work is in sight and people leave to join more active sites
- Less successful crews or individuals are let go first.
- The more difficult work may have been left to the end, may be more congested, or otherwise require only the skills of those brought in initially.
- Pressure to complete "the last few percent" dies down as management attention turns to more critical work.
- Latent defects may surface upon final inspection requiring re-work with no added measurable product to show for the effort.

A report issued by the National Electrical Contractors Association (NECA) in 1983 further illustrates the general shape of the resource loading S-curve. Data was collected from 40 different contractors on 54 building projects in 32 cities. The projects represented four broad types of public buildings competitively bid and which the contractors felt were typical of their business. The report includes the supporting data which show the ranges of variation.

Figure 2 shows the overall average manpower consumption rate S-curve for all the data collected (NECA 1983). The figure also shows the overall low and high values and it is interesting to note that the range of variation over a considerable number of projects is only 10% of the total time scale. It should also be noted that a particular condition typically prevails in electrical work on building construction. At the outset, only a small crew is required for installing conduit and other electrical hardware during the course of work by other trades. The bulk of the electrical work cannot be undertaken until those trades are substantially complete. In other words, the work takes longer to open up and accounts for a longer Stage 1 in this particular S-curve.

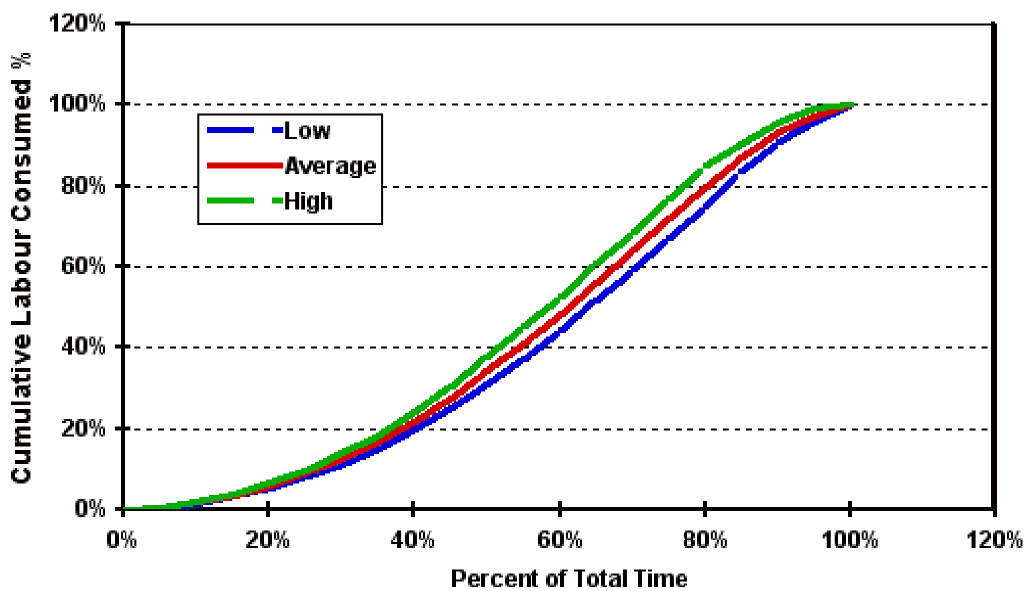

Figure 2 – Cumulative manpower consumed for electrical systems installation in new buildings — NECA

Progress S-curves (output)

As might be expected, the foregoing factors have a considerable impact on total production especially as represented by the more familiar output or progress S-curves.

A complete determination of the project status and projections to final completion for management action can perhaps best be tracked by an integrated cost/schedule system or technique known as "Earned Value and Performance Measurement" (Kerzner 1989). The earned value, i.e. the Budgeted Cost of Work Performed (BCWP), is determined at regular intervals during the course of the project. At the same time, the Actual Cost of Work Performed (ACWP) is also determined, and both are compared to the baseline plan which is the Budgeted Cost of Work Scheduled (BCWS). By presenting these results graphically as S-curves, the variances in cost and schedule can readily be seen, and by analyzing the results relative to the baseline plan S-curve, estimates can be made of anticipated variations at completion. The key elements of the technique are shown in

Figure 3.

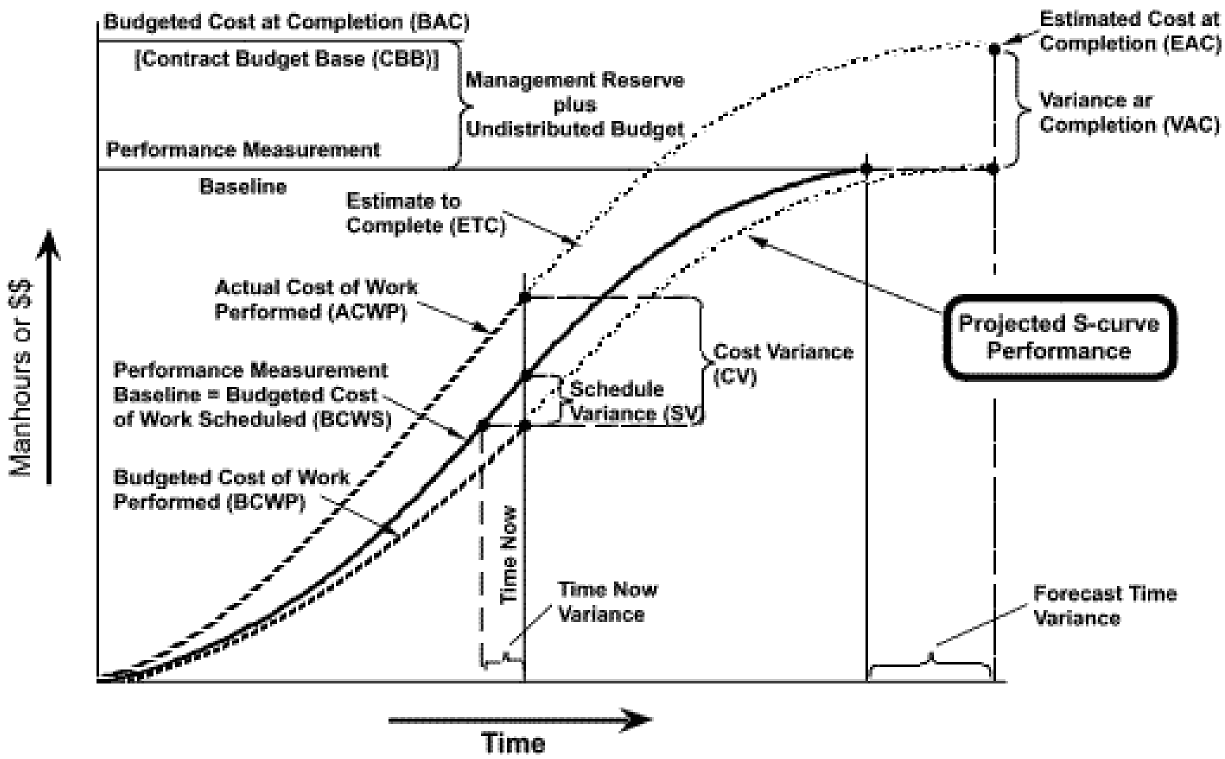

Figure 3 – Earned value and performance measurement

The technique is especially useful on projects involving a large number of significant activities by different trades and/or under conditions which change during the course of the work. There are, however, several weaknesses in the approach (Meredith 1985). Cost data must be collected which reflects the actual progress of the work, work in progress must be measurable and it must be measured. Consequently, this form of tracking requires significant additional effort or qualified dedicated staff to collect reasonably reliable data. This is particularly true where large purchases of off-site equipment may involve staged payment assessments.

Since the results are at best estimates of work-in-hand and the final results are estimated projections, the technique is not usually considered worth the effort on most projects. The exceptions are large complex projects, or projects on which this approach is required under the terms of the contract. When the technique is adopted, an essential element in its successful use is a realistically shaped baseline plan S-curve.

A better strategy for tracking progress is to identify the major critical activities on site that are measurable and plot those S-curves as surrogates for the whole job or stages of the job. Figure 4 shows three S-curves[2] illustrating different major activities. To facilitate comparison they are shown plotted as percent progress against percent time.

Curve (a) shows progress on a 5560 pipe pile driving contract lasting 137 working days. Curve (b) shows the cumulative progress on a 180,000 cu. yd. bulk excavation contract lasting 15 weeks. Notice the progressive addition of plant as the work opens up in the beginning, and the subsequent removal of plant as the availability of work runs out

2 From the author's personal records of progress tracking.

towards the end. Curve (c) shows progress on a 7-month civil contract as reflected by the approved monthly measurement progress billings.

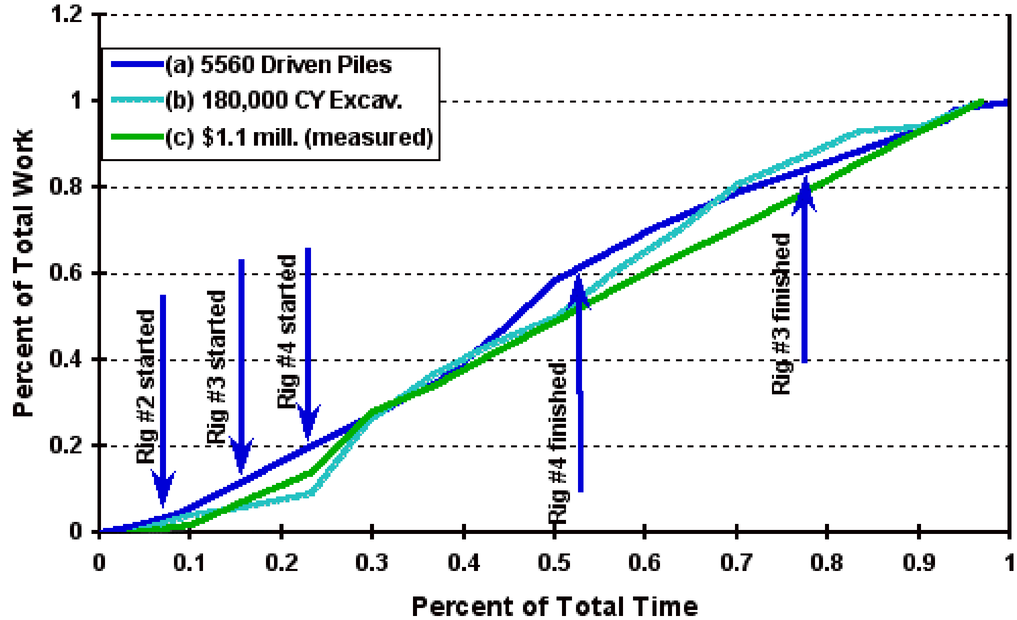

Figure 4 – Three examples of progress S-curves

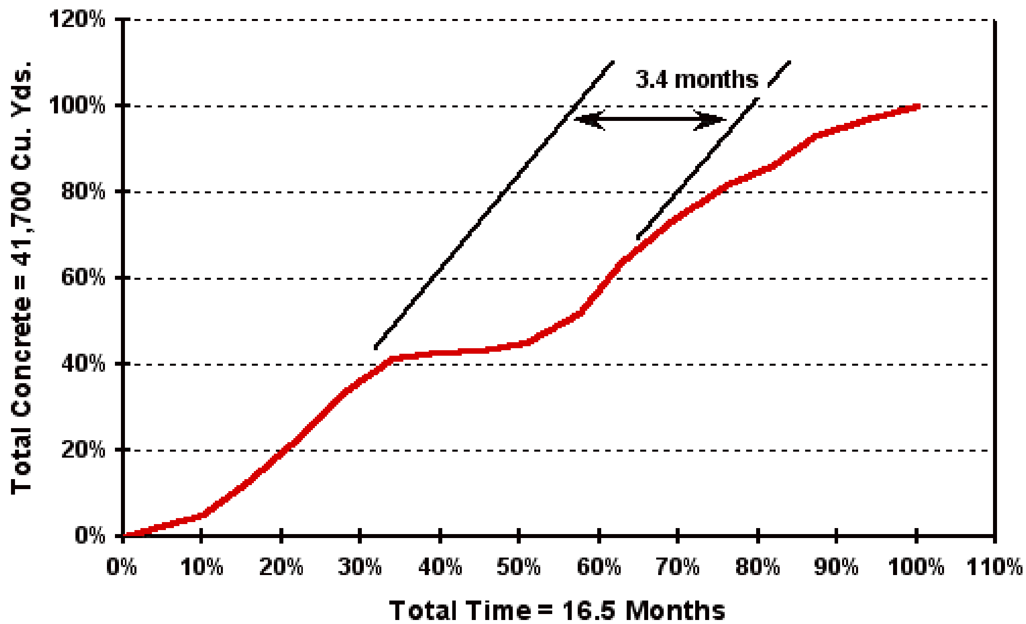

Figure 5 – Placing of formed structural concrete.
Total concrete = 31 900 m^3; total time = 16.5 months

Figure 5 shows measured progress on a 42,000 cu. yd. structural concrete activity of 16-months duration in Ontario, Canada. This curve is interesting because it clearly shows the slow down in progress over the winter months. The original data indicates that the virtual cessation of activities due to cold weather was only two-and-a-half months. However, due to the S-curve effects just before and after the cold weather cessation, the total impact of this condition was closer to three-and-a-half months. In the preparation of the original construction schedule, this situation could have been reasonably foreseen and an appropriate adjustment made to the "standard" S-curve profile. What can be Learned of Practical Value?

Manpower Consumption

In Figure 6, the data in Figure 1 has been re-plotted to a horizontal time scale of 100% and a vertical scale such that the overall average manpower loading is at 100%. Superimposed is a smoothed envelope curve representing the same data. This curve is in the shape of an asymmetrical dome and, since it is directly related to the shape of the manpower loading curves discussed earlier, also appears to be quite typical. The typical fit is never perfect of course, but it is suggested that the fit is sufficiently close to draw some conclusions relating to planning and management of similar type jobs.

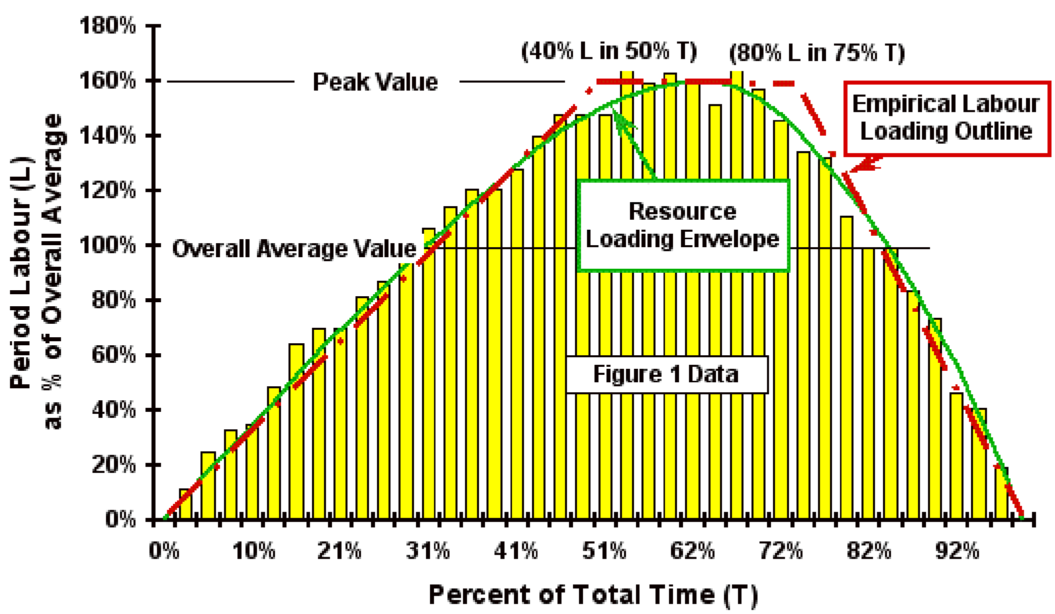

Figure 6 – Histogram, envelope, and empirical resource loading input of the Figure 1 civil contract example of site manpower

However, the mathematics of such a curve is complex and not particularly useful for preliminary planning purposes. A simple profile made up of straight lines would be more useful as a first approximation. Such a relationship has been suggested by Allen.

A First Approximation to Manpower Loading (Empirical Relation #1)

Allen puts forward the following simple empirical relationship as a first approximation to planned manpower loading (Allen 1979).

1. The maximum on-the-job manpower is 160% of the average manpower

requirement.
2. The maximum on-the-job manpower first occurs after 40% of the total manpower requirement has been expended.
3. The period of maximum on-the-job manpower accounts for 40% of the total manpower requirement.
4. The maximum on-the-job manpower first occurs when 50% of the project time has elapsed.
5. The period of maximum on-the-job manpower occurs for 25% of the project time.

Note that manpower may be measured in man-hours or dollars.

The resulting figure is a trapezoid and for convenience will be referred to as a "Standard Resource Input" (SRI) profile. This profile is also shown in Figure 6. Summarizing, it will be noted that 40% of the total manpower requirements occurs in the first 50% of the time, a further 40% of the total manpower requirements occurs in the next 25% of the time, and the last 20% of the manpower requirements occurs in the last 25% of the time.

The period of peak loading at 160% of the overall average is a valuable indicator. Once the total man-days and duration of the work have been estimated, the level of site support services required for the work force during the period of peak production can be determined.

For comparison, this SRI profile is shown in Figure 7 superimposed over the NECA manpower envelope corresponding to the NECA S-curve shown in Figure 2.

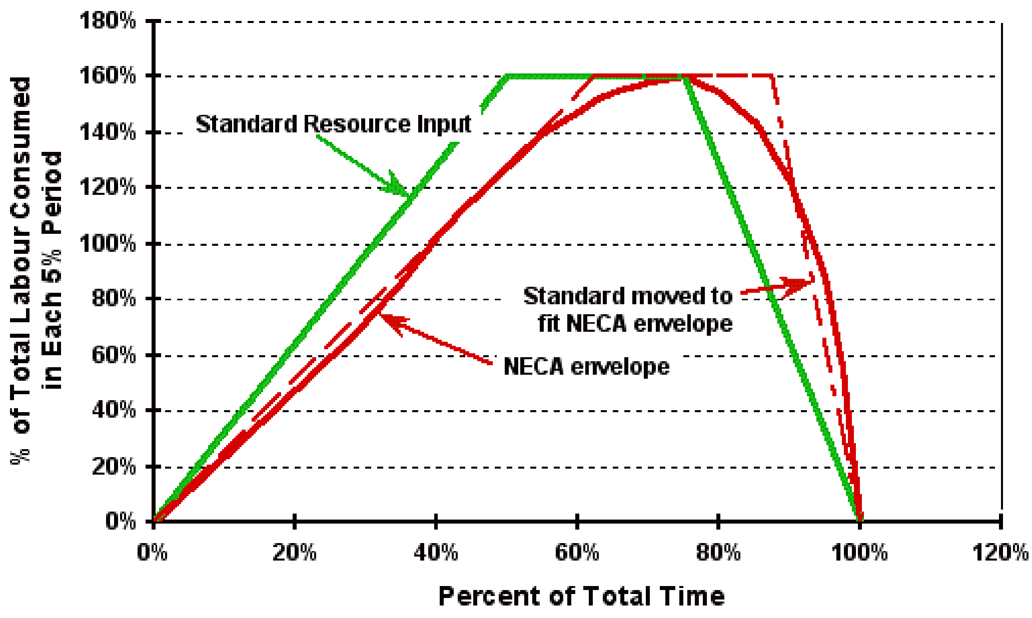

Figure 7 – Standard resource input vs. typical manpower loading of electrical systems installation in new building construction — NECA

It will be seen that the profile is very similar, but that the peak electrical manpower loading occurs some 10-12% later than in the SRI profile. This is due to the longer Stage 1 for the reasons described earlier. The SRI trapezoidal profile can be integrated to produce the cumulative total as shown by the S-curve marked as curve (a) in Figure 8.[3] Thus, this S-curve is made up of two quadratics and a middle linear portion. This curve will be referred to as the SRI S-curve.

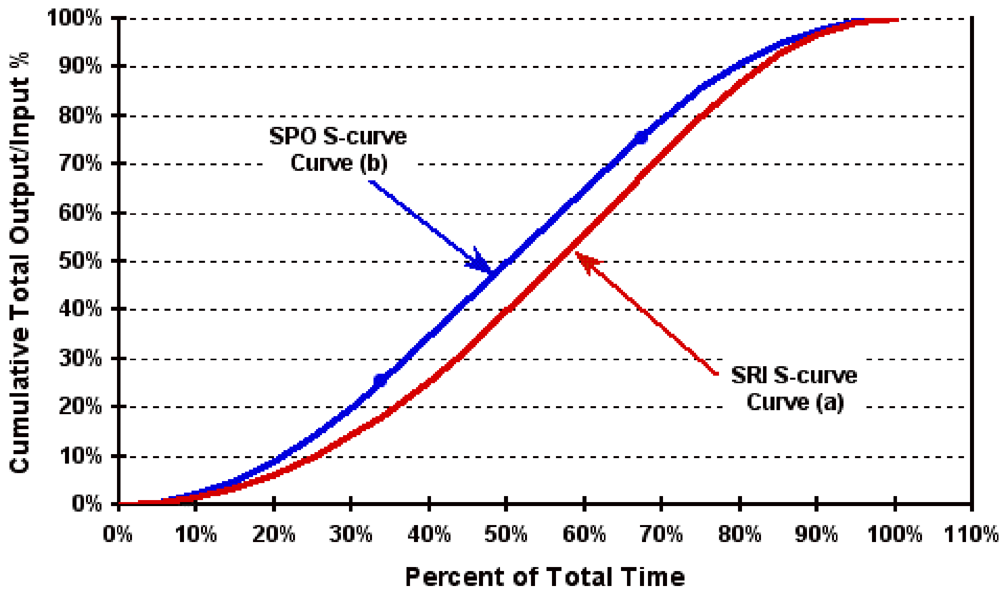

Figure 8 – Comparison of S-curves:
standard production output (SPO) vs. standard resource input (SRI)

Production S-curves in Practice

As described earlier, the typical S-curve effectively consists of three stages namely, Build-up, Steady-state and Run-down. Stage 1 is in fact the most critical since it is during this stage that the unique conditions of the site are experienced, and the stage is set for a steady, productive, and profitable run at the main body of the work. Stage 2 is important because peak productivity and efficiency must be attained and maintained without interruption for profit to be actually generated. Stage 3 is important for ensuring that the work is brought to an effective and satisfactory conclusion without time and money being wasted.

For example, if the manpower is cut too early the work gets extended. If it is cut too late unnecessary cost is incurred in paying for unproductive man-hours.

Since Stage 1 is the most critical to the subsequent successful conclusion of the work, so it is worth examining this stage more closely. As will have been gathered from the earlier descriptions, the shape of the S-curve in this stage is made up of two components.

a) The build-up of resources
b) Added production through productivity improvement

Build-up of resources has been discussed with examples in earlier sections.

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3 Formulae for the determination of points on the S-curves are given in Appendix 1

Added production through productivity improvement implies that the rate of output achieved will exceed that which might be inferred simply from examining the resource loading. On a well-run project this is a key management expectation, which will be reflected in the shape and timing of the progress S-curve for the job. Indeed, this leads to a second simple empirical relationship.

Empirical Relation #2

A First Approximation to a Project Progress curve[4]

A first approximation to project progress or output is suggested by the following empirical relationship.

- 25% of total progress is achieved in the first third of the total time,
- Another 50% in the next third, and
- The remaining 25% in the last third.

This, a curve representing an accelerating rate of progress will be exhibited in the first 25% of the time, while a similar but opposite curve will occur in the last third.

Like the SRI S-curve, this curve is also made up of two quadratics and a linear section in the middle. For convenience it will be referred to as a "Standard Production Output" (SPO) S-curve. Note that if this profile is being used for planning or forecasting, the 100% Total Time base will correspond to the realistically planned duration. If, however, the profile is being used for post completion analysis, the Actual Total Time may be substituted. The units of progress may be expressed in units appropriate to the work, such as excavation volumes, numbers of piles, or value of work produced as shown in Figure 4.

The SPO S-curves is shown plotted as curve (b) in Figure 8. The differences between the two curves is essentially developed during Stage 1 of the S-curves, i.e. the first third of the Total Time. The shapes of the two curves are largely driven by the addition of resources. However, if the difference is attributed to improvement through learning, it can be shown that this difference is equivalent to a Learning Curve of 86% based on the LL-CA Model calculation discussed below. This is within the range of productivity-improvement-through-learning observed through independent measurements on construction sites.

Productivity Improvement Through Learning

The theory behind productivity improvement through learning is worth reviewing briefly, because there are somewhat different, though similar, approaches to the mathematics involved. In addition, the theory can make a useful contribution in terms of:

- Demonstrating the importance of consistent management direction and effectiveness
- Showing the benefits of establishing on a job the highest possible degree of repetition
- Ensuring sufficient and continuous availability of work prior to commencement
- Underpinning work crew motivation and attitude
- Forecasting output, and hence time and cost to completion
- Estimating work which is comparable, but which may be significantly different in the

4 This empirical progress curve has been used on the job by the author for many years and has been offered to students in cost management and cost control project management workshops.

amount to be accomplished.

And conversely, in

- Demonstrating the adverse impacts of interference to the flow of work

Learning Curve vs. Experience Curve

The principle of the learning curve is given in Appendix 1. However, there is some confusion in the construction industry regarding the use of this term. Because different construction work typically takes place under unique conditions at a unique site, it is useful to differentiate between productivity improvement due to "learning" and that due to "experience".

As craft apprentices learn their trade, productivity increases. However, when skilled crafts perform a specific task on site and repeat it a number of times, there is a similar productivity increase. The former increase is due to "learning the skill", while the latter is due to acquiring "experience of the particular site conditions" associated with the work activity at the time.

This is an important distinction because it has significant implications if a site is ***not*** well run, or a job is ***subject to changes*** which interrupt the development of the learning pattern.

In construction, unfortunately, "learning curve" is typically used to refer to the productivity improvement resulting from the site experience, that is, by crafts who are already skilled at their trade. Consistent with this practice, this paper uses "learning curve" to imply "experience" and therefore assumes that all crafts have already acquired the relevant skills for the work.

Original Theory

The phenomenon of "learning" was first expressed mathematically in 1936 by T. P. Wright. He observed in the aircraft industry that certain costs per unit tend to decrease in a predictable pattern as the workers and their supervisors become more familiar with the work. These decreasing costs are a function of a learning process in which fewer and fewer man-hours are required to produce a unit of work as more and more units are produced. The key elements of the theory may be summarized as follows (Adrian 1987).

- The repetition of any task leads to an improvement in productivity as a result of the experienced gained.
- This phenomenon is well established in the mass production industry as well as in the construction industry under appropriate circumstances.
- The application of the theory (in the construction industry) assumes that operatives start with the necessary basic skills as well as the required support for the work to be accomplished.
- Productivity improvement then typically follows a constant ratio relationship which is expressed as follows.

For every doubling of units, the cumulative average time per unit is reduced by a constant ratio.

This relationship is illustrated in Table 1, showing examples of Cumulative Average Time per Unit (Chellew 1974).

Number of Units in Sequence	Cumulative Average Time per Unit	
	90% Ratio	80% Ratio
1	100.0	100.0
2	90.0	80.0
4	81.0	64.0
8	72.9	51.2
16	65.6	40.9
32	59.1	32.8

Table 1: - Examples of Cumulative Average Time per Unit for Two Different Ratios

In Table 1, the time taken for the first unit is 100%. At a 90% ratio, the average time taken for the first and second unit is 90%, i.e., the actual time taken for the second unit is 80%. By the time the fourth unit is reached the average time taken for all four units is 90%x90% = 81% and so on.

These values can be plotted as curves as shown in Figure 9. However, if the same data is plotted on log-log paper as shown in Figure 10, the result is a straight line which is more useful for manual analysis or mathematical illustration.

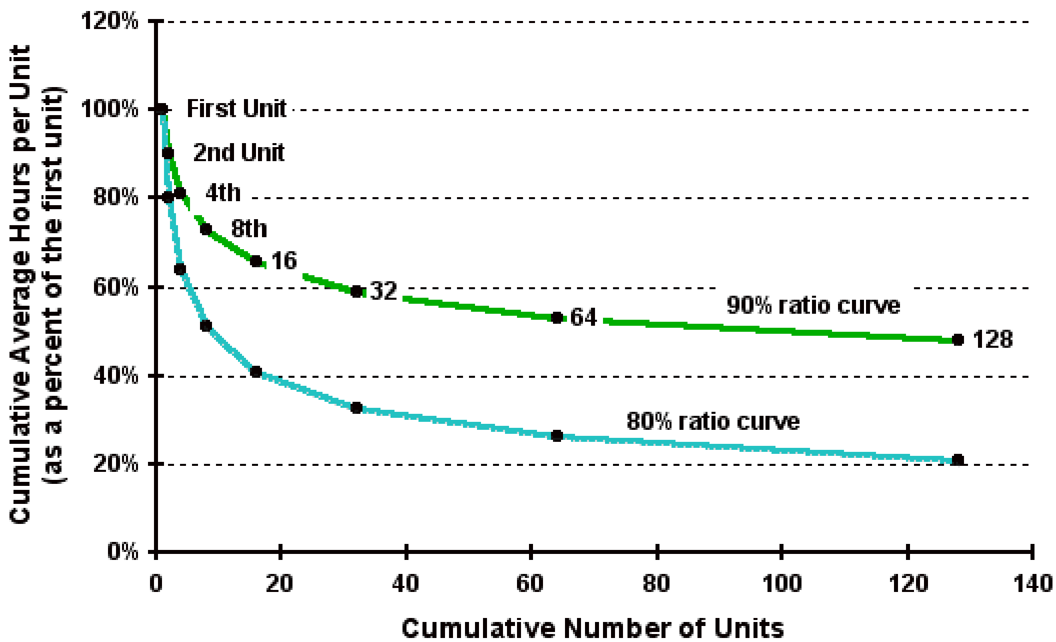

Figure 9 – Illustration of learning curves

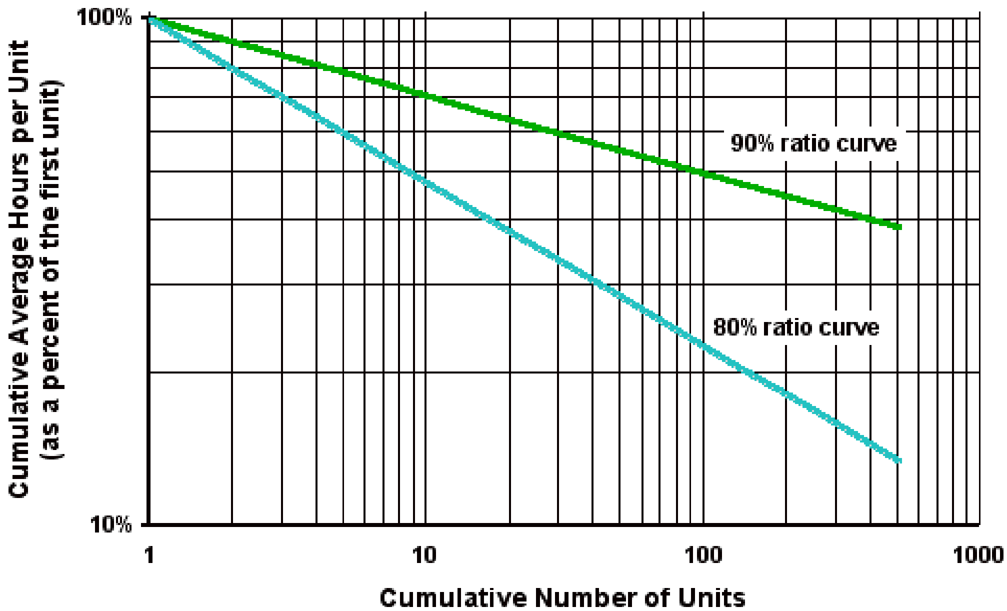

Figure 10 – Learning curves plotted on log-log scale

Two Approaches

The above log-log relationship can be expressed mathematically as follows.

> *The cumulative average time (or cost) for each of 'n' units up to the n^{th} unit, when plotted against the number of units on log-log paper, produces a straight line.*

This may be referred to as the “Log Linear - Cumulative Average Approach" (The LL-CA Model). This relationship is useful in forecasting or comparing similar operations but with significantly different numbers of units involved. It is also useful in analyzing large amounts of data as, for example, the records of a large number of units produced from a precasting yard. This is because the cumulative average curve has considerable power to smooth out the unit data. It can also be deceptive because this power increases as the quantity increases (Thomas 1986). It is, therefore, less useful for examining the expectations for individual units or the latest unit such as would be needed in tracking actual progress on a construction site.

This has led to a variation of the first relationship which states as follows.

> *The time (or cost) of the n^{th} unit, when plotted against the number of units on log-log paper, produces a straight line.*

This may similarly be referred to as the "Log Linear - Unit Approach" (The LL-U Model) (Drewin 1982; DSMC 1989). The mathematics of both models are developed and compared in Appendix 2. Table 2 shows calculations of the time ***to*** the nth unit and the time ***of*** the nth unit over a range from one to fifty units for ratios ranging from 70% to 95% as determined by each approach. The Cumulative Average figures are shown on white background, while the corresponding Cumulative Unit figures are shaded. As might be expected, the results of the two approaches are similar but not identical. The differences in results obtained from the two approaches vary from about 7% for a repetition of only five units at a 95% productivity ratio to over 100% for 50 units at a ratio of 70%.

Lp = r s=logr/log2	0.950 -0.074				0.900 -0.152				0.850 -0.234			
	Cum-Av		Cum-Unit		Cum-Av		Cum-Unit		Cum-Av		Cum-Unit	
n	Tn/U1	Un/U1	Tn/U1	U'n/U1	Tn/U1	Un/U1	Tn/U1	U'n/U1	Tn/U1	Un/U1	Tn/U1	U'n/U1
1	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000
5	4.4	0.829	4.7	0.888	3.9	0.675	4.4	0.783	3.4	0.538	4.2	0.686
10	8.4	0.784	9.0	0.843	7.0	0.602	8.1	0.705	5.8	0.452	7.3	0.583
15	12.3	0.760	13.2	0.818	9.9	0.565	11.5	0.663	7.9	0.409	10.1	0.530
20	16.0	0.743	17.2	0.801	12.7	0.540	14.8	0.634	9.9	0.382	12.6	0.495
25	19.7	0.731	21.2	0.788	15.3	0.521	17.9	0.613	11.8	0.362	15.0	0.470
30	23.3	0.721	25.1	0.777	17.9	0.507	20.9	0.596	13.5	0.346	17.3	0.450
35	26.9	0.713	29.0	0.769	20.4	0.495	23.9	0.583	15.2	0.334	19.6	0.434
40	30.4	0.705	32.8	0.761	22.8	0.485	26.7	0.571	16.8	0.323	21.7	0.421
45	34.0	0.699	36.6	0.755	25.2	0.476	29.6	0.561	18.4	0.314	23.8	0.410
50	37.4	0.694	40.3	0.749	27.6	0.469	32.4	0.552	20.0	0.307	25.8	0.400

Lp = r s=logr/log2	0.800 -0.322				0.750 -0.415				0.700 -0.515			
	Cum-Av		Cum-Unit		Cum-Av		Cum-Unit		Cum-Av		Cum-Unit	
n	Tn/U1	Un/U1	Tn/U1	U'n/U1	Tn/U1	Un/U1	Tn/U1	U'n/U1	Tn/U1	Un/U1	Tn/U1	U'n/U1
1	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000	1.0	1.000
5	3.0	0.418	3.9	0.596	2.6	0.314	3.7	0.513	2.2	0.224	3.4	0.437
10	4.8	0.329	6.6	0.477	3.8	0.230	5.9	0.385	3.1	0.152	5.2	0.306
15	6.3	0.287	8.8	0.418	4.9	0.193	7.6	0.325	3.7	0.123	6.6	0.248
20	7.6	0.261	10.8	0.381	5.8	0.171	9.2	0.288	4.3	0.105	7.8	0.214
25	8.9	0.242	12.6	0.355	6.6	0.155	10.5	0.263	4.8	0.094	8.8	0.191
30	10.0	0.228	14.3	0.335	7.3	0.144	11.8	0.244	5.2	0.085	9.7	0.174
35	11.1	0.217	16.0	0.318	8.0	0.135	13.0	0.229	5.6	0.078	10.5	0.160
40	12.2	0.208	17.5	0.305	8.7	0.127	14.1	0.216	6.0	0.073	11.3	0.150
45	13.2	0.200	19.0	0.294	9.3	0.121	15.1	0.206	6.3	0.069	12.0	0.141
50	14.2	0.193	20.5	0.284	9.9	0.116	16.1	0.197	6.7	0.065	12.7	0.134

= Cum-Av Approach   = Cum-Unit Approach

Table 2 - Comparison of Cum. Av. and Cum. Unit Productivity from 70% to 90%

In practice, one would select one approach or the other depending on the objective, and use the corresponding set of ratios. It does mean, however, that

> *When comparing the learning ratios on different jobs or of different crews on similar work, the method of calculation must be the same and it must be specified.*

Illustration of Learning Curve Application

For purposes of illustration, consider the following hypothetical case. The construction of floors on a 25 storey concrete high-rise building are being tracked. From the second floor up, all floors are virtually the same, so that the second floor is the first of a uniform series of 24. The roof and mechanical penthouse are not included in the observations.

Construction data is collected as follows.[5]

Time sheets are carefully marked up with job allocations, and hours are abstracted for forming and pouring concrete on each standard floor. The man-hours for the first in the series is noted as 1175 man-hours. The second, third and fourth in the series take 855, 905, and 735 respectively. This data is plotted on log-log paper using the LL-U Model as shown by line (a) in Figure 11. At this stage the data suggests a line whose slope is -.152 (approx. 90% learning ratio) and that future floors would be expected to take the times shown in Column 1b of Table 3.

	Col 1a	Col 1b	Col 2a	Col 2b	Col 3
Floor#	**Observed 4 floors**	**Projected 90% learning**	**Observed 6 floors**	**Projected 85% learning**	**Final All floors**
1	1175		1175		1175
2	880		880		880
3	940		940		940
4	820		820		820
5		822	700		700
6		800	690		690
7		781		698	620
8		765		676	700
9		752		658	695
10		740		642	720
11		729		628	650
12		720		615	620
13		711		604	680
14		703		593	670
15		696		584	710
16		689		575	660
17		683		567	640
18		677		559	670
19		671		552	750
20		666		546	710
21		661		540	850
22		656		534	790
23		652		528	935
24		648		523	1060
Projected Totals:-		**18036**		**15825**	
Final Total:-					**18,335**

Table 3 – High-rise repetitive construction: hypothetical case

However, suppose actual records for the next two floors, five and six, produce results of 700 and 690 respectively. The addition of the latest data suggests a new line whose slope is -.234 (approx. 85%) as shown by line (b) in Figure 11, and the new times taken to complete are as shown in Table 3, Column 2b. The new result shows a reduction in total hours of approximately 2200 hours (12%, or the equivalent of some four extra floors).

5 The variation of “actuals” selected for the illustration are within the authors experience.

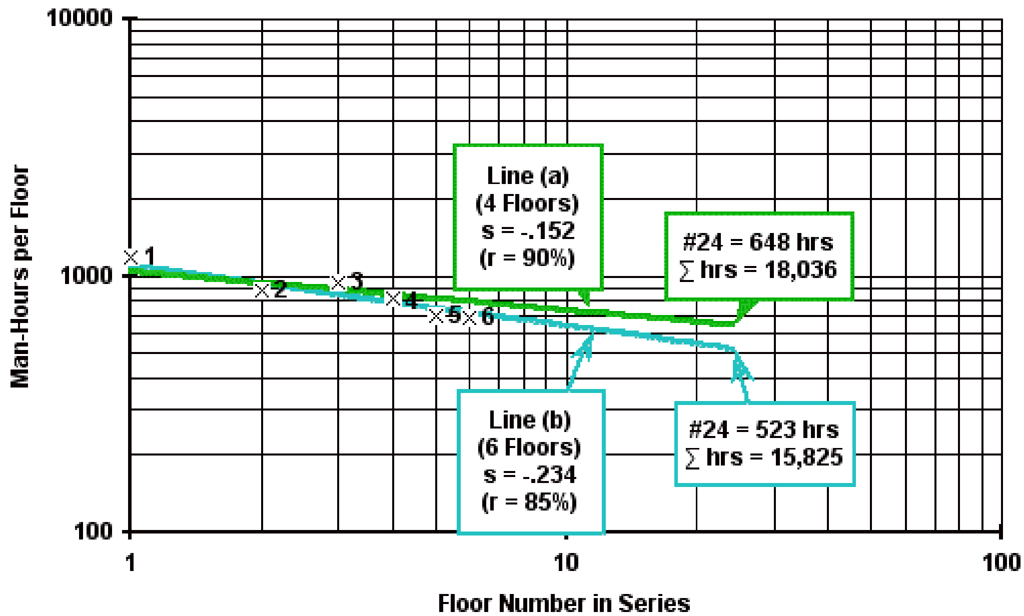

Figure 11 – High-rise repetitive construction: four floors projected (green), and six floors projected (turquoise)

Typically, practical reality follows neither of the two models. When record keeping is continued until all floors are completed, the results could be as shown in Table 3, Column 3. These results are shown plotted in Figure 12.

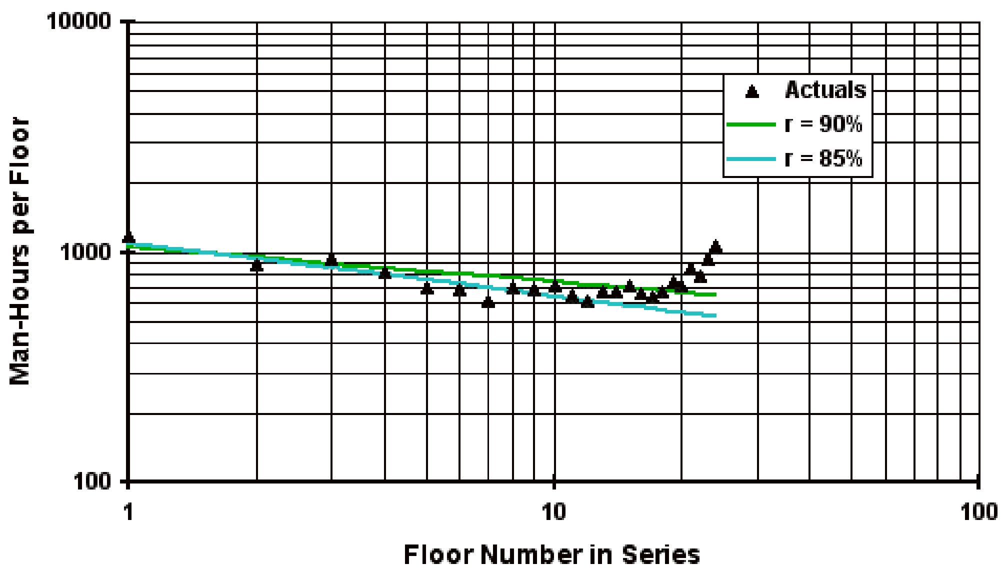

Figure 12 – High-rise repetitive construction: cumulative unit projections and observed (LL-U model)

Many projects experience a decrease in productivity at the end of a run of work, (Barrie, Paulson 1978) and in the example the "tail end" departs significantly from either of the two earlier projections. The total man-hours shown in Table 3, Column 3 is 15% higher than the second projected total in column 2b.

The same data, plotted according to the LL-CA Model, are shown in Figure 13. It will be seen that this model substantially conceals the significant changes in trends associated with the "tail end" effect. Thus, the LL-U Model, although not consistent with the original theory, is a more useful tool in many practical applications and for project management observation and control.

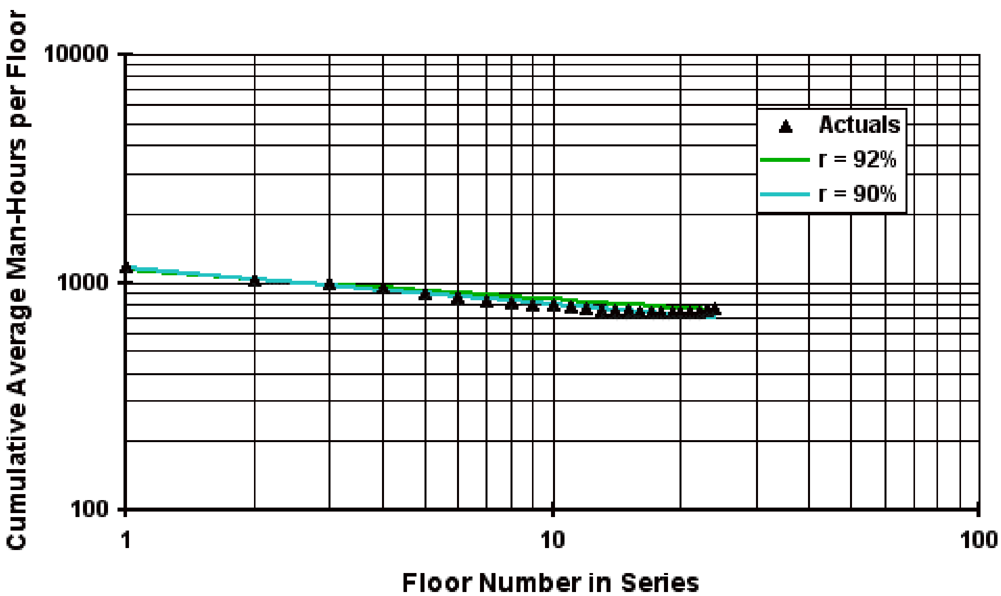

Figure 13 – High-rise repetitive construction: cumulative average projections and observed (LL-CA model)

When the figures shown in Table 3, Column 3 are plotted at normal scales, they display a shape sometimes referred to as the "Bath Tub" effect as shown in Figure 14. In fact, this is simply a reflection of some of the considerations associated with each of the three stages of the S-curve discussed in an earlier section.

This suggests that the application of "Learning Curve Theory" on a construction site should be limited to the first 25% or so of the total production under consideration, which is to say approximately 30-35% of the allotted time. In the high-rise construction example, the target for reaching optimum performance must be the 6th or 7th floor.

Issues Regarding Total Time and Stage 1 Time

A reasonable question to ask is how can the planner be assured of choosing the right overall time (i.e., equivalent to 100%) and why should the learning always take 30-35% of that value? Should it not be possible to contemplate a 36-storey high-rise, rather than 24, and still achieve the Stage 2 efficiency of the 24 storey high-rise in the first six floors?

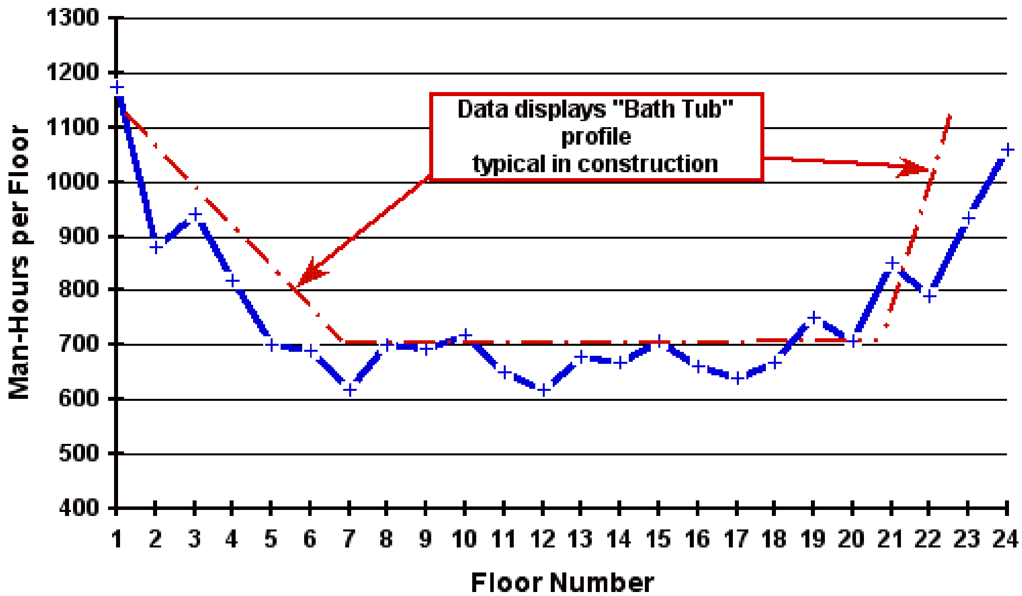

Figure 14 – High-rise repetitive construction: showing the "bath tub" effect

The practical reality is that if the building is that much larger in all likelihood the whole scale of the project operation is correspondingly larger and will be planned and organized accordingly. This includes increased use of temporary materials, plant, equipment and site organization all optimized to suit the larger project.

The planned time must also be realistic and achievable, especially if it is being compressed. Having chosen this time, it is essential that all the supporting logistics of the site, including management, supervision, equipment, supplies etc., are all present to support this choice. Failing this, it is the author's observation that the job then "takes on a will of its own" wherein it charts its own progress record. Thus, it is the organizational culture associated with the site that ultimately determines the final outcome.

As one example of what can go wrong, Figure 15 compares actual production of rock excavation with planned production on a less than well managed site preparation contract.[6] Partly due to changes, final quantities were significantly higher than originally anticipated, the planned peak level of production was never reached, and Stage 1 of the S-curve took more time. Not surprisingly in this case, the whole contract took a lot longer to complete — and ended in litigation.

Conclusions

A review of resource input and production output on construction work shows three separate stages in any activity. This is true whether viewed at the task, trade, subcontract or whole project level. These stages constitute "build-up", "steady-state" and "run-down". Each stage has distinctive features.

6 From the author's personal records of an actual contract.

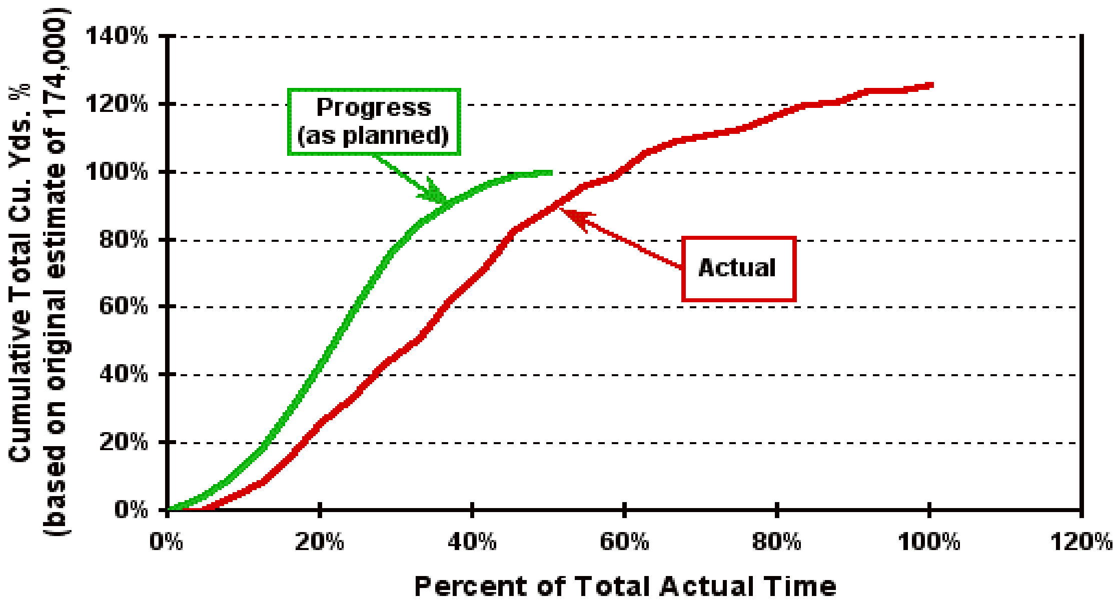

Figure 15 – Comparison of actual vs. planned production of bulk rock excavation. Percent cumulative total is based on original estimate of 133 000 m^3 (174,000 cubic yards)

If data over these three stages are viewed as a histogram of period resources input over the duration of the work, a first approximation empirical profile can be articulated. That is: 40% of resource input occurs in the first 50% of the time, a further 40% input in the next 25% of the time and the remaining 20% in the last 25% of the time. This profile determines that peak loading will be 160% of the overall average.

If the same data is plotted as a running total on a percentage of total scale on both axes, the result is a typical S-curve. On a well-run project actual timing of this peak loading, i.e., Stage 2, appears to vary by only 10-15%.

As can be expected, production output follows a similar profile. However, if input and output S-curves are plotted to the same scales, the output S-curve will precede the input S-curve to the extent that productivity improvement is achieved. For the whole of this work to be optimized, it appears that productivity improvement must essentially be completed in Stage 1.

An empirical output or progress S-curve is suggested. This takes the form of one quarter of the progress in the first third of the time, another half in the next third and the final quarter in the final third of the time. A realistic productivity improvement ratio of 86% in Stage 1 would account for the difference between the two empirical S-curves of output and input.

Obviously, the best source of information for planning and estimating is derived from experience of very similar previous work. In the absence of specific experience, however, these empirical relations can be used as a first approximation, particularly for early planning.

Many construction projects offer various opportunities for repetitive work, though the total number of repetitions may be small compared to manufacturing processes. However, when carefully managed and tracked, such work provides distinct opportunities for productivity improvement. To optimize productivity gain, management energy must be focused on the first 25% of the series. The target must be to hit peak production within one-third of the planned total time.

Two approaches to productivity improvement calculations are described. The first focuses on the Cumulative Average Time for 'n' units. However, the second, a modification of the first but focusing on the time taken for the n^{th} unit, is more useful in most construction applications. In any case, it is suggested that the learning curve theory should not be carried further into the work than the first 25-30%.

Application of S-curve theory to construction work includes comparative estimating, forecasting, and quantifying the effects of delays upon performance. In these, the natural loss of productivity in the final 25% of the work should also not be over looked.

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Appendix 1

Definitions and Formulae adopted in this paper

Production (Rate)
The rate at which units are produced over a given period of time, independent of the number of man-hours consumed.

Productivity
In its broadest form, productivity may be described as a measure of how well the resources in a firm are brought together and used to accomplish a set of results.[7] In its simplest form, it may be expressed as the ratio of output to input or the actual rate of output or production per unit of time worked.[8] These represent measures of production efficiency. When measurements are taken over a given period of time, the period productive efficiency is the number of units produced in that time period divided by the number of man-hours to do so.

S-curve
When the cumulative total of on-going work on a construction job is plotted against time, the resulting curve typically follows the shape of the letter "S". It is more generally referred to as a Progress Curve.

Learning or Experience Curves
Studies have shown that the change in cost associated with a change in productivity has, in many situations, a characteristic curve that can be estimated with reasonable accuracy. This is called the “learning curve” or “experience curve”.[9]

The underlying phenomenon is that skill and productivity in performing tasks improve with experience and practice and there are a number of different ways of plotting this relationship that facilitate mathematical analysis. Two models of Learning Curves are given in Appendix 2, Learning Curve Mathematics.

Well-run
A well-run construction job implies that adequate and realistic planning has taken place and a positive cultural environment has been established for the performance of the work on site. It also means that supporting logistics, including delivery of materials and equipment, have been properly assessed and will be provided when needed to enable optimized crew sizes to maximize their production at least cost at each point in time. It follows that the resulting project should be perceived as successful in terms of meeting requirements and being completed within credible time and cost parameters. For an owner this would mean that the resulting facility has satisfied the stipulated needs, within reasonable time and budget. For a contractor, it would mean satisfying the owner at a profit.

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7 Cleland, D. I. 1990. Project Management Strategic Design and Implementation. TAB Books Inc. Blue Ridge Summit, Pennsylvania. 344.

8 Cleland, D. I. and Kerzner H. 1985. A Project Management dictionary of Terms. Van Nostrand Reinhold C. New York. 193.

9 Anthony, R. N. and Reece, J. S. 1975. Management Accounting: Text and Cases. Richard D. Irwin, Inc., Homewood, Illinois. 540.

In contrast, actual progress on a not-so-well-run job will depart from the plan or proceed as "a voyage of discovery". The records will likely reflect wasted manpower before sufficient work is available or after it is substantially completed, lower productivity, higher manpower turnover, additional learning costs, added supervision, labour and non-labour-related job expenses and overhead, added material storage, handling and wastage, and extended completion.

Standard Resource Input curve (SRI S-curve)

Points on the SRI S-curve may be determined as follows:

In Stage 1: If t_1 is a time between 0 and 50% then the cumulative total production is

$$p = 0.5\ t_1^2 \times 1.6/0.5$$
$$= 1.6 \times t_1^2$$

In Stage 2: If t_2 is a time between 50% and 75%, then the cumulative total production is

$$p = p_{50} + 1.6\ (t_2 - t_{50})$$

or

$$p = 0.40 + 1.6\ (t_2 - 0.50)$$
$$= 1.6\ t_2 - 0.40$$

In Stage 3: If t_3 is a time between 75% and 100% then the cumulative total production is

$$p = p_{75} + 0.8\ (t_{100} - t_{75}) - 0.8\ (t_{100} - t_3)^2 / (t_{100} - t_{75})$$

or

$$p = 0.8 + 0.8\ (1 - 0.75) - 0.8\ (1 - t_3)^2 / (1 - 0.75)$$
$$= 1 - 3.2\ (1 - t_3)^2$$

Standard Production Output curve (SPO S-curve)

Points on the SPO S-curve may be determined as follows:

In Stage 1: If t_1 is a time between 0 and 33.3% then the cumulative total production is

$$p = 0.5\ t_1^2 \times 1.5/0.333$$
$$= 2.25\ t_1^2$$

In Stage 2: If t_2 is a time between 33.3% and 66.7%, then the cumulative total production is

$$p = p_{33} + 1.5\ (t_2 - t_{33})$$

or

$$p = 0.25 + 1.5\ (t_2 - 0.333)$$
$$= 1.5\ t_2 - 0.25$$

In Stage 3: If t_3 is a time between 66.7% and 100% then the cumulative total production is

$$p = p_{67} + 0.75\ (t_{100} - t_{67}) - 0.75\ (t_{100} - t_3)^2 / (t_{100} - t_{67})$$

or

$$p = 0.75 + 0.75 (1 - 0.667) - 0.75 (1 - t_3)^2 / (1 - 0.667)$$
$$= 1 - 2.25 (1 - t_3)^2$$

Appendix 2

Learning Curve Mathematics

Learning Curve Mathematics is based on the observation that when a particular task or sequence of work is repeated without interruption certain costs per unit tend to decrease in a predictable pattern. This is attributed to the experienced gained as the workers and their supervisors become more familiar with the work being repeated. There are, however, two approaches or mathematical models for purposes of practical application.

Model A: Log Linear - Cumulative Average (LL-CA)

This model was first stated mathematically in 1936 by T. P. Wright who observed that productivity improvement typically follows a constant ratio relationship in the form

> *For every doubling of units, the cumulative average time per unit is reduced by a constant ratio.*

When plotted to log-log scale, the result is a straight line. Expressed more fully

> *The cumulative average time (or cost) for each of 'n' units up to the n^{th} unit, when plotted against the number of units on log-log paper, produces a straight line.*

This is referred to as the Log Linear - Cumulative Average (LL-CA) Model.

Suppose the time for the first unit is taken as 100% and the ratio 'r' is 80%. then the average time for the first and second units is 80%. That is the second unit took 60% of the time of the first unit. By the time the fourth unit is reached, the average time taken for all four units is 64% and so on.

Consider Figure (a) and the following relationships.

Symbols

Let C_n = y ordinate = Cumulative Average Time over 'n' units (CAT_n)

n = x ordinate = number of units

N = n.th unit

U_1 = Time to produce first unit (constant) = T_1

U_n = Time to produce n.th unit

s = Slope of CAT line on log-log plot

r = the constant ratio (by definition), otherwise known as the Learning Curve Ratio; Learning Rate; Decremental Rate of Experience; Learning percent; Percent Learning etc. Usually expressed as 80%, 90% etc, and

s', r', = corresponding symbols for Model B

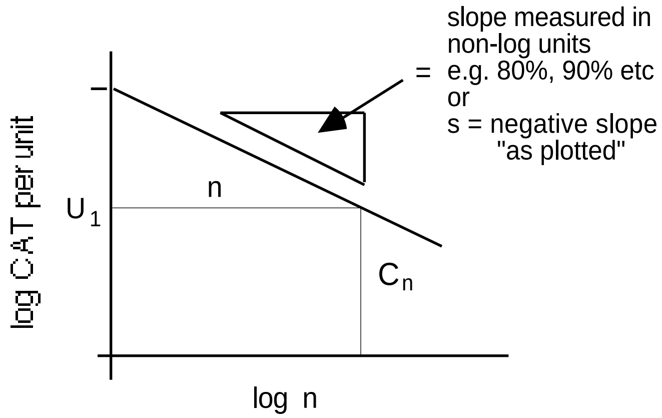

Figure A1: Typical learning curve plotted on a Log-Log scale

From Figure A1

$$\log C_n = \log U_1 + s \cdot \log n$$

or $C_n = U_1 \cdot n^s$ (Note the form $y = a \cdot x^b$) .....(1)

By definition

$$r = \frac{C_{2n}}{C_n} = \frac{U_1 \cdot (2n)^s}{U_1 \cdot n^s} = 2^s$$

(Where "r" = % Learning Curve)

$$\therefore \quad s = \frac{\log r}{\log 2}$$ .....(2)

T_n = Time *up to* the *n* th unit (i.e. time to complete *n* units)

$$= n \cdot C_n = n \cdot U_1 \cdot n^s = U_1 \cdot n^{(s+1)}$$ .....(3)

Time *for* the *n* th unit $= U_n = U_1 \cdot \left\{ n^{(s+1)} - (n-1)^{(s+1)} \right\}$ .....(4)

For purposes of plotting n against T_n (Learning Curve):

1) Select any value of r (ratio)

2) Calculate $s = \dfrac{\log r}{\log 2}$

3) From $Tn = U_1 \cdot n^{(s+1)}$, calculate $\dfrac{T_n}{U_1}$ for values of n

4) If required to "fit" a particular point:

calculate $T_N \times T_n$

Model B: Log Linear - Unit (LL-U)

Model A is useful in forecasting or comparing similar operations but with significantly different numbers of units involved. It is less useful for examining results for individual units as in tracking progress on a construction site. This has led to a variation which is expressed as follows.

> *The time (or cost) of the n^{th} unit, when plotted against the number of units on log-log paper, produces a straight line.*

This is referred to as the Log Linear - Unit (LL-U) Model.

Again, assume that the time for the first unit is taken as 100% and the ratio 'r' is 80%. In this model, the reduction of time between the first and second unit will be 20%. Between the 2nd and 4th units it will be a further 20% and so on. That is to say, the fourth unit will take 64% of the time of the first unit. This is very different from Model A in which the fourth unit must take about 48% of the first unit.

In Figure (a) the 'y' ordinate is now 'time per unit' (rather than Cumulative Average Time per Unit). Using the symbols previously listed, the new relationship is expressed by

$$U'_n = U'_1 \cdot n^{s'} \qquad .....(5)$$

By definition

$$r = \frac{U'_{2n}}{U'_n} = \frac{U'(2n)^{s'}}{U'(n)^{s'}} = 2^{s'}$$

$$\text{or} \quad s' = \frac{\log r'}{\log 2} \qquad .....(6)$$

and

$$T_n = \text{Time up to end of the } n \text{ th unit} = U'_1 + \int_1^n \left(U'_1 \cdot n^{s'}\right) \cdot dn$$

Integrating

$$= U'_1 + \left[\frac{U'_1 \cdot n^{(s'+1)}}{s'+1} + k\right] - \left[\frac{U'_1}{s'+1} + k\right]$$

or

$$T_n = U'_1\left[1 + \frac{n^{(s'+1)}}{s'+1} - \frac{1}{s'+1}\right] \qquad .....(7)$$

and

$$\frac{T_n}{U'_1} = 1 + \left[\frac{n^{(s'+1)}}{s'+1} - \frac{1}{s'+1}\right] \qquad .....(8)$$

In the special case where T_1 is equated to 100%, then U'_1 equals 1

$$T_n = 1 + \left[\frac{n^{(s'+1)}}{s'+1} - \frac{1}{s'+1} \right] \qquad .....(9)$$

Plotting Stage 1 of the Standard Progress Output (SPO) S-curve

In this case Model A is easier to use and the difference from Model B is minimal. By definition of the SPO S-curve, the learning curve must "force fit" to the start of Stage 2, which is one quarter of the units at one third of the time and at the slope of Stage 2 which, between points N and N+1, is two thirds.

Let T_n and N = the point at which the S-curve joins the start of Stage 2
Let T_n = the point to be plotted but where $n < N$

Then from equation (3) above

$$\frac{T_n}{T_N} = \frac{U_1 \cdot n^{(s+1)}}{U_1 \cdot N^{(s+1)}}$$

or

$$T_n = T_N \cdot \left(\frac{n}{N} \right)^{(s+1)} \qquad .....(10)$$

In the special case where 100% of the units are being plotted against 100% of the time, the learning curve ratio can be calculated by trial and error to 71% or $s \cong -0.5$.

Thus

$$T_n = 33.3\% \left(\frac{n^{-.5+1}}{25\%^{-.5+1}} \right) = 0.333 \left(\frac{n^{0.5}}{0.25^{0.5}} \right)$$

or

$$T_n = 0.67\sqrt{n} \qquad .....(11)$$

This curve is shown plotted as Stage 1 in figure 8.